

*Erratum***Erratum to: Classical  $\mathcal{W}$ -Algebras and Generalized Drinfeld–Sokolov Hierarchies for Minimal and Short Nilpotents****Alberto De Sole<sup>1</sup>, Victor G. Kac<sup>2</sup>, Daniele Valeri<sup>3</sup>**<sup>1</sup> Dipartimento di Matematica, Sapienza Università di Roma, P.le Aldo Moro 2, 00185 Rome, Italy.  
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There is an error in Sect. 6.2 of the original article in the computation of equations of the generalized Drinfeld–Sokolov hierarchy associated to the minimal nilpotent element  $f$  of the Lie algebra  $\mathfrak{g} = \mathfrak{sl}_n$ ,  $n \geq 3$ , corresponding to the unique (up to a constant factor) non-zero central element  $c \in \mathfrak{g}_0^f$  (such  $c$  exists for minimal  $f$  only in the case  $\mathfrak{g} = \mathfrak{sl}_n$ ,  $n \geq 3$ ).

Namely, equation  $\frac{dL}{dt_1} = 0$  in the second line of page 652 should be replaced by

$$\frac{dL}{dt_1} = \sum_{k \in J_{\frac{1}{2}}} \left( \psi([f, v_k]) \psi([c, [f, v^k]]) \right)'.$$

Consequently, one should add to Eq. (6.17) of the original article the following equations

$$\begin{aligned} \frac{d\psi(u)}{dt_{\bar{1}}} &= \psi([c, u])'' - \frac{1}{2(x|x)} L\psi([c, u]), \\ \frac{dL}{dt_{\bar{1}}} &= \sum_{k \in J_{\frac{1}{2}}} \left( \psi([f, v_k]) \psi([c, [f, v^k]]) \right)'. \end{aligned} \quad (1)$$

This error affected the example of  $\mathfrak{g} = \mathfrak{sl}_3$  in Sect. 8 of our paper [DSKV14a]. Namely, to (8.7) one should add two conserved densities:

$$g_{\bar{0}} = \varphi, \quad g_{\bar{1}} = 6\psi_+ \psi_- ,$$

to Eq. (8.9) one should add the equation

$$\frac{d}{dt_0} \begin{pmatrix} L \\ \psi_+ \\ \psi_- \\ \varphi \end{pmatrix} = \begin{pmatrix} 0 \\ -3\psi_+ \\ 3\psi_- \\ 0 \end{pmatrix},$$

to Eq. (8.10) one should add equations

$$\begin{aligned} \frac{dL}{dt_1} &= 6(\psi_+\psi_-)', \\ \frac{d\psi_{\pm}}{dt_1} &= \mp 3\psi_{\pm}'' \pm 3L\psi_{\pm} \mp \varphi^2\psi_{\pm} - \frac{3}{2}\psi_{\pm}\varphi' - 3\varphi\psi_{\pm}', \\ \frac{d\varphi}{dt_1} &= 0, \end{aligned}$$

and to Eq. (8.20) one should add equations

$$\frac{d}{dt_0} \begin{pmatrix} L \\ \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 0 \\ -3\psi_+ \\ 3\psi_- \end{pmatrix}, \quad \frac{d}{dt_1} \begin{pmatrix} L \\ \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 6(\psi_+\psi_-)' \\ -3\psi_+'' + 3L\psi_+ \\ 3\psi_-'' - 3L\psi_- \end{pmatrix}.$$

The latter is the well known Yajima–Oikawa (YO) equation [YO76].

Thus, in [DSKV14a] we proved that the YO hierarchy is obtained by Dirac reduction from the minimal  $\mathfrak{sl}_3$  generalized Drinfeld–Sokolov hierarchy, and, as a result, we gave in formulas (8.4) and (8.5) two compatible Poisson structures for the YO hierarchy. The latter were found in [Che92].

Furthermore, this error affected the example considered in Sect. 2.5 of our paper [DSKV14b]: one should add Eqs. (1) to the equations at the end of Sect. 2.5 there in the case of  $\mathfrak{g} = \mathfrak{sl}_n$ ,  $n \geq 3$ . Thus, in Sect. 2.5 of [DSKV14b] we proved that the  $n - 2$ -component YO hierarchy is obtained by Dirac reduction from the minimal  $\mathfrak{sl}_n$  generalized Drinfeld–Sokolov hierarchy. As a result, we gave two compatible Poisson structures for the  $n - 2$ -component YO hierarchy, which we denoted by  $\overline{H}_0$  and  $\overline{H}_1^D$  in Sect. 2.5 of [DSKV14b].

The new type of reduced Kadomtsev–Petviashvili (KP) hierarchy, called the constrained KP hierarchy, was introduced in [KSS91, KS92], where it was observed that the YO hierarchy can be obtained as a constrained KP hierarchy (and this was used in [Che92] to find its bi-Poisson structure). A more general  $s$ -vector constrained KP hierarchy was introduced in [SS93], where it was shown that the multi-component YO hierarchy, studied in [Ma81], can be obtained by this construction. After that, in [ZC94] the more general  $s$ -vector  $m$ -constrained KP hierarchy was introduced (the constrained KP hierarchy of [KS92, Che92, SS93] corresponds to  $m = 2$ ).

Our main observation in this regard is that the  $s$ -vector  $m$ -constrained KP hierarchy is isomorphic to the Dirac reduction by conformal weight 1 fields of the generalized Drinfeld–Sokolov hierarchy [DSKV14a, DSKV13], associated to the Lie algebra  $\mathfrak{g} = \mathfrak{sl}_{m+s}$  and its nilpotent element  $f$  corresponding to the partition  $(m, 1, \dots, 1)$  of  $m + s$ . (Note that for both hierarchies the number of fields is equal to  $m - 1 + 2s = \dim(\mathfrak{g}^f/\mathfrak{g}_0^f)$ .) For  $m = 2$  this observation is proved in [DSKV14b].

We are grateful to Professor Takayuki Tsuchida, who pointed out to us that our Eq. (8.10) in [DSKV14a] is a higher symmetry of the YO equation in the YO hierarchy [Che92], which led us to the discovery of the error in the original article.

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